

Solution

Claim: There exists an optimal answer such that each color's edges are a connected subgraph of the tree.

Proof: Consider an optimal tree coloring and let it be rooted from an arbitrary vertex. Define top_c the closest vertex to the root of that subgraph for each connected subgraph of the tree. Among all the connected subgraphs of the tree that don't have a unique color, choose one with the farthest top_c . If there isn't any, then the claim is satisfied. Otherwise, it's easy to see that top_c can't be the root of the tree. Now change the color of every edge of c to the color of the edge between top_c and its parent. You can see the answer doesn't decrease and our tree remains good.

The above claim makes the problem so easy. We can find the answer using dynamic programming. Let $dp_{v,i,j}$ be the maximum number of colors we can use in the subtree of v such that the first longest path with different colors that starts somewhere in subtree of v and reaches v has at most i edges, and the second-longest path same as the first one that has different colors with the first one has at most j edges and sum of these two doesn't exceed $k - 1$. Notice that when we find values of these dynamic programming, the answer is $dp_{root,k-1,k-1}$.

We can evaluate $dp_{v,i,j}$ from v 's children by considering different cases for each child. Let u be a child of v .

- $dp_{u,j-1,j-1} + 1$ if we color edge vu with a new color. This path can be the second longest path.
- $dp_{u,i-1,i-1} + 1$ if we color edge vu with new color. This path can be the first longest path, so we have to care that at most one of the children update with this.
- $dp_{u,k-1,i-1}$ if we color edge vu with the color of above edge in its longest path. This subtree has a path that can be the first longest path, so we have to care that at most one of the children uses this much color.
- $dp_{u,k-1,j-1}$ if we color edge vu with the color of above edge in its longest path.

Just keep in mind that for getting full score, you have to decrease the memory usage.