Solution

In this tutorial, n is the number of vertices, m is the number of edges, and c is the number of available pencils.

First of all, we can show that the number of groups is equal to the number of groups that occur after decomposing edges of the tree to maximal connected trees with the same edge color. In this decomposition, each edge appears once, but a vertex can appear more than once.

Lets define some variables, G is the number of groups, V is the set of vertices, C is the set of colors, t_v is the number of different colors connected to vertex v, n_i is the number of vertices in tree i, and m_i is the number of edges in tree i.

It's easy to see that every vertex will appear t_v times in groups, and $m_i + 1 = n_i$ in each group, each edge appears once so $\sum_{i=0}^{G} m_i = m$, and also $\sum_{i=0}^{G} n_i = \sum_{v \in V} t_v$.

The number of groups is equal to $\sum_{i=0}^{G} n_i - m_i = \sum_{v \in V} t_v - m$, so instead of calculating the required expectation we can calculate $(\sum_{v \in V} t_v) - m = (\sum_{v \in V}(\sum_{c \in C} p_{v,c})) - m$, wich $p_{v,c}$ is the probability that vertex v has at least one connected vertex of color c.

If vertex v has degree d_v then probability of having a particular color is $1 - (c - 1/c)^{d_v}$.

So, the answer is equal to $(\sum_{v \in V} c * (1 - (c - 1/c)^{d_v}) - m$. By this formula, we can solve the problem in linear time by preprocessing $(c - 1/c)^i$ for each i.