

Problem PI

The Pentagon Conjecture

Time limit: 10 seconds

Memory limit: 1024 megabytes

Problem Description

Counting the number $k(H, G)$ of copies of a designated subgraph H in a given undirected simple graph G efficiently can be challenging. For example, counting $k(C_3, G)$ for an n -node graph G may require $\Omega(n^{3-\delta})$ time for any constant $\delta > 0$ if your algorithm is “combinatorial.” By C_k , we denote the simple cycle of length k . When the designated subgraph H becomes more complicated, the running time required to count $k(H, G)$ usually grows very quickly.

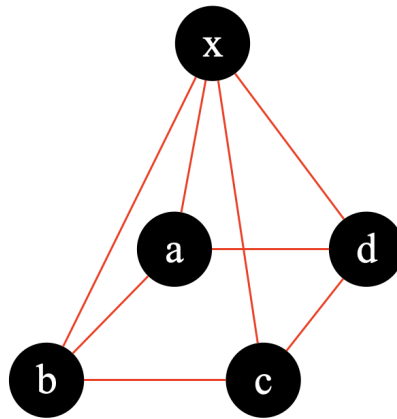


Figure 1: A pyramid P is an undirected simple graph consisting of 5 nodes and 8 edges, as depicted above. It is not hard to check that $k(C_3, P) = 4$, $k(C_4, P) = 5$, and $k(C_5, P) = 4$.

Bob is a researcher who needs to detect rare events in a given graph network. For example, in a random graph R of n nodes in which each edge is included with probability $1/2$, $k(C_3, R) = \binom{n}{3}/8$ in expectation. If he finds that $k(C_3, R)$ deviates significantly from the expectation, he may conclude that R is unlikely to be a random graph generated by the above process. Bob is studying the correlation between $k(C_3, G)$ and $k(C_5, G)$ for n -node graphs. There is a conjecture saying that, for any n -node undirected simple graph G , if

$$k(C_3, G) \geq \frac{5}{4} \left(n^{1.5} + \frac{n}{2} \right),$$

then $k(C_5, G) > 0$. Your task is to implement an efficient algorithm that verifies whether the conjecture is false.

Input Format

The first line contains exactly one integer t indicating the number of testcases. Then the testcases follow. For each testcase, the first line contains exactly two integers n and m , separated

by a space. n denotes the number of nodes in the given undirected simple graph G , and m denotes the number of C_3 s given in the subsequent lines. The set of edges in G is exactly the union of the edges in the given C_3 s. Each of the subsequent m lines contains exactly three distinct integers x , y , and z , separated by a space, where x , y , and z denote a distinct C_3 in G with end-nodes at x , y , and z . The nodes in G are numbered from 1 to n .

Output Format

For each testcase, output the 5 node identifiers of any pentagon in G in order on a line if $k(C_5, G) > 0$ (i.e. output a set of 5 nodes $x_1x_2x_3x_4x_5$ so that (x_i, x_{i+1}) is an edge for each $1 \leq i \leq 4$ and (x_1, x_5) also is an edge; if there are multiple solutions, output any of them); or otherwise output “-1” on a line.

Technical Specification

- $1 \leq t \leq 10$.
- $1 \leq n \leq 10^4$.
- $m = \lceil 5/4(n^{1.5} + (n/2)) \rceil$.

Sample Input 1

```
1
7 28
1 2 3
1 2 4
1 2 5
1 2 6
1 2 7
1 3 4
1 3 5
1 3 6
1 3 7
1 4 5
1 4 6
1 4 7
1 5 6
1 5 7
1 6 7
2 3 4
2 3 5
2 3 6
2 3 7
2 4 5
2 4 6
2 4 7
2 5 6
2 5 7
2 6 7
3 4 5
3 4 6
3 4 7
```

Sample Output 1

```
1 2 3 4 7
```