Problem PG
A Packing Problem

Time limit: 1 second
Memory limit: 1024 megabytes

Problem Description

Jade is having fun playing around with items and boxes. She is wondering whether or not the items she has can be packed into the boxes. The scenario is described as follows.

Jade has m boxes $\mathcal{B} := \{ B_1, B_2, \ldots, B_m \}$ that are nonidentical to each other. Despite the nonidentical appearances of the boxes, they have a uniform size T. On the other hand, she has n items $\mathcal{I} := \{ I_1, I_2, \ldots, I_n \}$, where item I_j has size a_j that is either s_1 or s_2 for some s_1, s_2. In other words, $a_j \in \{s_1, s_2\}$ for all $1 \le j \le n$. Furthermore, it is known that $0 < s_1, s_2 \le T$ and $s_1, s_2 \notin (T/4,\ 3T/4)$. That is, either $s_i \le T/4$ or $s_i \ge 3T/4$ for $i \in \{1, 2\}$.

In Jade's rule of thumb for packing the items, not every item can be placed in every box. In particular, she has associated each item I_j with a subset $P_j \subseteq \mathcal{B}$ which denotes the set of boxes in which item I_j is allowed to be placed. To be precise, item I_j can be placed in box B_i if and only if $B_i \in P_j$. For convenience, for any $B_i \in \mathcal{B}$, also define $P_i^{-1} := \{ I_j \in \mathcal{I} :\ B_i \in P_j \}$ to be the set of items that are allowed to be placed in box B_i.

Under this scenario, Jade is wondering, whether or not it is possible to pack all the items in the boxes such that the total size of the items in each box is at most T. In other words, Jade is interested in knowing the existence of an assignment function $\sigma\colon \mathcal{I} \mapsto \mathcal{B}$ such that

- For any j with $1 \le j \le n$, $\sigma(I_j) = B_i$ only if $B_i \in P_j$.
- For any i with $1 \le i \le m$,
$$\sum_{I_j \in \sigma^{-1}(B_i)} a_j \ \le\ T.$$

In particular, if there exists no such assignment, then we say that the box size T is infeasible.

As a known fact, one way to certify the infeasibility of any $T \ge 0$ in the above scenario is to demonstrate a set of variables α_j and β_i for all $I_j \in \mathcal{I}$, $B_i \in \mathcal{B}$ such that the linear constraints listed below in LP-(t) with $t := T$ is satisfied.

In other words, there exists a set of valid estimations on the sizes of the items and boxes in the sense that, whenever the total size of an item combination C is at most the size of a box B_i, so is their total estimated sizes. Furthermore, the total estimated item sizes are strictly larger than the total estimated sizes of the boxes.

While knowing the above fact and feeling that the box size T may not be feasible for the items,

$$\sum_{1\le j\le n} \alpha_j > \sum_{1\le i\le m} \beta_i \qquad \text{LP-(t)}$$

$$\sum_{I_j\in C} \alpha_j \le \beta_i, \qquad \text{for any } 1\le i\le m \text{ and any } C\subseteq P_i^{-1} \text{ such that } \sum_{I_j\in C} a_i \le t.$$

Jade has a hard time finding such a set of variables. One day, Jade's best friend, Mike, dropped by and said

> The instance you give is very hard to pack! Why don't you try to prove that the box size $(\,1+3/4\,)\cdot T$ is feasible?

While it is easier to use boxes with enlarged size $(\,1+3/4\,)\cdot T$, Jade insists that each box must not contain more than one item with size strictly larger than $T/4$!

Provided the above information, your task in this problem is to help Jade compute either

1. An assignment σ for the enlarged box size $(\,1+3/4\,)\cdot T$ such that no box contains more than one item with size strictly larger than $T/4$, or
2. A set of α_j and β_i that satisfies LP-(t) with $t := T$ which shows that the box size T is infeasible for the items.

Input Format

The first line contains two integers n and m, which denote the number of items and the number of boxes. Then there are n lines, each of which describes the parameters for the n items. In particular, the j^{th} line starts with two integers a_j and p_j, the size of I_j and the cardinality of P_j. Then p_j integers follow, which denote the indexes of the boxes in P_j. The last line contains a single integer T.

You may assume that the indexes of the boxes are numbered from 1 to m.

Output Format

Depending on the resulting outcome, the output format is different.

If an assignment for box size $(\,1+3/4\,)\cdot T$ is found, then print in the first line the string "Assignment". Print in the second line n integers which denotes the indexes of the boxes in which the n items are placed. If there are multiple answers, print any of them.

On the other hand, if a set of α_j and β_i that satisfy LP-(t) with $t := T$ is found, then print in the first line the string "Proof". Print the values of α_j for all $1\le j\le n$ and β_i for all $1\le i\le m$ in two lines separately. If there are multiple solutions, print any of them that satisfies the following two conditions.

- α_j and β_i are non-negative integers for all $1 \le j \le n$ and $1 \le i \le m$.
- $\max\left(\max_{1 \le j \le n} \alpha_j,\ \max_{1 \le i \le m} \beta_i \right) \le 3 \times 10^5$.

Technical Specification

- $1 \le n \le 100$, $1 \le m \le 100$.
- T is a multiple of 4. Furthermore, $1 \le T \le 10^5$.
- For both $i \in \{1, 2\}$, either $1 \le s_i \le T/4$ or $3T/4 \le s_i \le T$ must hold.
- $a_j \in \{s_1, s_2\}$ for all $1 \le j \le n$.

For the output, the following conditions must be satisfied.

- α_j and β_i are non-negative integers for all $1 \le j \le n$ and $1 \le i \le m$.
- $\max\left(\max_{1 \le j \le n} \alpha_j,\ \max_{1 \le i \le m} \beta_i \right) \le 3 \times 10^5$.

Sample Input 1

```
3 3
4 2 1 2
4 2 2 3
4 2 3 1
4
```

Sample Output 1

```
Assignment
1 2 3
```

Sample Input 2

```
4 3
4 2 1 2
4 2 2 3
4 2 3 1
4 3 1 2 3
4
```

Sample Output 2

```
Proof
1 1 1 1
1 1 1
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