



# Problem J

## Strongly Matchable

Time Limit: 3 Seconds

Let  $G$  be a simple undirected graph with  $n$  vertices, whose vertex and edge sets are denoted by  $V(G)$  and  $E(G)$ , respectively. Two edges of  $G$  are said to be *adjacent* if they share a common vertex. Similarly, two vertices of  $G$  are said to be *adjacent* if they share a common edge, in which case the common edge joins the two vertices; an edge and a vertex on that edge are called *incident*. A subset  $M$  of  $E(G)$  is called a *matching* of  $G$  if no two edges in  $M$  are adjacent;  $M$  is called a *perfect matching* if every vertex of  $G$  is incident to exactly one edge of  $M$ . So, a matching  $M$  of  $G$  is perfect if and only if  $|M| = \frac{n}{2}$ .

The existence of a perfect matching in  $G$  can be decided in polynomial time, thanks to a polynomial-time algorithm for finding a maximum matching, a matching that contains the maximum number of edges. Besides, there are two more interesting problems on the existence of a perfect matching in  $G$ :

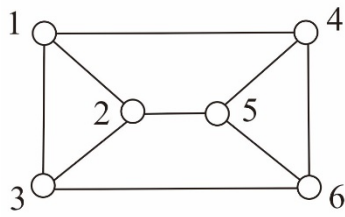
- Given a partition of  $V(G)$  into  $S$  and  $T$  with  $|S| = |T| = \frac{n}{2}$ , does  $G$  has a perfect matching in which every edge joins a vertex in  $S$  and a vertex in  $T$ ?
- For every partition of  $V(G)$  into  $S$  and  $T$  with  $|S| = |T| = \frac{n}{2}$ , does  $G$  has a perfect matching in which every edge joins a vertex in  $S$  and a vertex in  $T$ ?

From the well-known Hall's marriage theorem, we can derive a condition that characterizes the existence of a required perfect matching for the first question as follows: Let  $G'$  be the spanning subgraph of  $G$  with the edges joining vertices both in  $S$  or both in  $T$  being deleted, i.e.,  $V(G') = V(G)$  and  $E(G') = \{(u, v) \in E(G) \mid \text{either } u \in S \text{ and } v \in T \text{ or } v \in S \text{ and } u \in T\}$ . Then,  $G$  has a required perfect matching between  $S$  and  $T$  if and only if  $G'$  has a perfect matching. Moreover, the Hall's theorem leads to that  $G'$  has a perfect matching if and only if  $|N(X)| \geq |X|$  for every subset  $X$  of  $S$ , where  $N(X)$  denotes the neighborhood of  $X$ , i.e., the set of all vertices in  $T$  adjacent to some vertex of  $X$ . The question, of course, can be answered in polynomial time, also thanks to a maximum matching algorithm that runs in polynomial time.

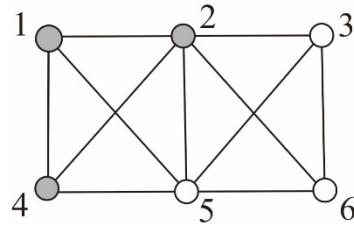
Is there an efficient algorithm to answer the second question? A graph that admits a positive answer for the second question is called *strongly matchable*; that is, a graph  $G$  is *strongly matchable* if  $G$  has a perfect matching in which each edge joins two vertices, one in  $S$  and the other in  $T$ , for every partition of  $V(G)$  into  $S$  and  $T$  with  $|S| = |T| = \frac{n}{2}$ . For example, the graph shown in Figure J.1 (a) is strongly matchable because there is a perfect matching for each of the three partitions up to symmetry:  $M = \{(1,4), (2,5), (3,6)\}$  for  $S = \{1,2,3\}$  and  $T = \{4,5,6\}$ ;  $M = \{(1,3), (2,5), (4,6)\}$  for  $S = \{1,2,4\}$  and  $T = \{3,5,6\}$ ;  $M = \{(1,3), (2,5), (6,4)\}$  for  $S = \{1,2,6\}$  and  $T = \{3,4,5\}$ . However, the graph of (b) is not strongly matchable because there is no perfect matching between  $S = \{1,2,4\}$  and  $T = \{3,5,6\}$ . Your job is to write an efficient running program for deciding whether or not an input graph with an even number of vertices is strongly matchable.

### Input

Your program is to read from standard input. The first line contains two positive integers  $n$  and  $m$ , respectively, representing the numbers of vertices and edges of the input graph, where  $n$  is even,  $n \leq 100$ , and  $m \leq \frac{n(n-1)}{2}$ . It is followed by  $m$  lines, each contains two positive integers  $u$  and  $v$  representing an edge between the vertices  $u$  and  $v$  of the input graph. It is assumed that the vertices are indexed from 1 to  $n$ .



(a)



(b)

Figure J.1: The graph shown in (a) is strongly matchable, but the graph of (b) is not.

**Output**

Your program is to write to standard output. Print exactly one integer in a line. If the input graph is strongly matchable, the integer should be 1; otherwise, the integer should be -1.

The following shows sample input and output for two test cases.

**Sample Input 1**

```
6 9
1 4
4 6
6 3
3 1
1 2
3 2
4 5
6 5
2 5
```

**Output for the Sample Input 1**

```
1
```

**Sample Input 2**

```
6 11
1 2
2 3
3 6
6 5
5 4
4 1
2 5
1 5
2 4
2 6
3 5
```

**Output for the Sample Input 2**

```
-1
```