

2025 Canadian Computing Olympiad
Day 1, Problem 3
Balanced Integer

Time Limit: 30 seconds

Problem Description

Since the CCO often uses integers, Alice needs to learn about the integers! A positive integer n can be written in base b as the sequence $d_{m-1}d_{m-2}\dots d_1d_0$ if the following hold:

- Each digit d_i is between 0 and $b - 1$, inclusive.
- $d_{m-1} > 0$.
- $n = d_{m-1} \times b^{m-1} + d_{m-2} \times b^{m-2} + \dots + d_1 \times b^1 + d_0 \times b^0$.

For example, the integer 2025 in base 19 is the sequence (5, 11, 11) because $2025 = 5 \times 19^2 + 11 \times 19^1 + 11 \times 19^0$.

An integer n is b -balanced if, when n is written in base b , the average of the digits is $\frac{b-1}{2}$.

For example, 2025 is 19-balanced because $\frac{5 + 11 + 11}{3} = 9 = \frac{19 - 1}{2}$.

Alice can easily find integers that are 19-balanced. However, she has trouble finding integers that are balanced in multiple ways. Given B and N , please help Alice find the minimum integer x such that:

- x is b -balanced, for all $2 \leq b \leq B$.
- $x \geq N$.

Input Specification

The first line of input contains two space-separated integers B and N ($N \geq 1$).

It is guaranteed that the answer does not exceed 10^{18} .

The following table shows how the available 25 marks are distributed:

Marks Awarded	Bounds on B	Bounds on N
2 marks	$2 \leq B \leq 7$	$1 \leq N \leq 10^4$
6 marks	$2 \leq B \leq 6$	$N = 10^{10}$
2 marks	$2 \leq B \leq 7$	None
9 marks	$8 \leq B \leq 11$	$N = 1$
4 marks	$B = 8$	None
2 marks	$9 \leq B \leq 11$	None

Output Specification

Output the minimum integer x from the problem statement.

Sample Input 1

4 100

Output for Sample Input 1

141

Explanation of Output for Sample Input 1

141 in base 2 is 10001101. The average digit is $\frac{1+0+0+0+1+1+0+1}{8} = 0.5 = \frac{2-1}{2}$. Therefore, 141 is 2-balanced.

141 in base 3 is 12020. The average digit is $\frac{1+2+0+2+0}{5} = 1 = \frac{3-1}{2}$. Therefore, 141 is 3-balanced.

141 in base 4 is 2031. The average digit is $\frac{2+0+3+1}{4} = 1.5 = \frac{4-1}{2}$. Therefore, 141 is 4-balanced.

Lastly, $141 \geq 100$.

Sample Input 2

7 10000000000

Output for Sample Input 2

16926961207710

Hint

Feel free to use these code snippets as part of your solution.

```
// Important: If x is 0, the result is undefined.
int base_2_length(unsigned long long x) {
    return 64-__builtin_clzll(x);
}

int base_2_sum(unsigned long long x) {
    return __builtin_popcountll(x);
}
```